11-1 Arithmetic Sequences

Main Ideas

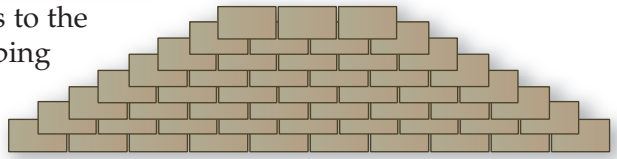
- Use arithmetic sequences.
- Find arithmetic means.

New Vocabulary

sequence
term
arithmetic sequence
common difference
arithmetic means

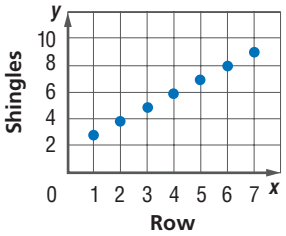
GET READY for the Lesson

A roofer is nailing shingles to the roof of a house in overlapping rows. There are three shingles in the top row. Since the roof widens from top to bottom, one more shingle is needed in each successive row.



Row	1	2	3	4	5	6	7
Shingles	3	4	5	6	7	8	9

Arithmetic Sequences The numbers 3, 4, 5, 6, …, representing the number of shingles in each row, are an example of a sequence of numbers. A **sequence** is a list of numbers in a particular order. Each number in a sequence is called a **term**. The first term is symbolized by a_1, the second term is symbolized by a_2, and so on. A sequence can also be thought of as a discrete function whose domain is the set of positive integers over some interval.



Many sequences have patterns. For example, in the sequence above for the number of shingles, each term can be found by adding 1 to the previous term. A sequence of this type is called an arithmetic sequence. An **arithmetic sequence** is a sequence in which each term after the first is found by adding a constant, called the **common difference**, to the previous term.

Study Tip

Sequences
The numbers in a sequence may not be ordered. For example, the numbers 84, 102, 97, 72, 93, 84, 87, 92, … are a sequence that represents the number of games won by the Houston Astros each season beginning with 1997.

EXAMPLE **Find the Next Terms**

1 **Find the next four terms of the arithmetic sequence 55, 49, 43, … .**

Find the common difference d by subtracting two consecutive terms.

$49 - 55 = -6$ and $43 - 49 = -6$ So, $d = -6$.

Now add -6 to the third term of the sequence, and then continue adding -6 until the next four terms are found.

$$43 \xrightarrow{+(-6)} 37 \xrightarrow{+(-6)} 31 \xrightarrow{+(-6)} 25 \xrightarrow{+(-6)} 19$$

The next four terms of the sequence are 37, 31, 25, and 19.

CHECK Your Progress

1. Find the next four terms of the arithmetic sequence $-1.6, -0.7, 0.2, \ldots$.

It is possible to develop a formula for each term of an arithmetic sequence in terms of the first term a_1 and the common difference d. Consider the sequence in Example 1.

Sequence	numbers	55	49	43	37	…	
	symbols	a_1	a_2	a_3	a_4	…	a_n
Expressed in Terms of d and the First Term	numbers	$55 + 0(-6)$	$55 + 1(-6)$	$55 + 2(-6)$	$55 + 3(-6)$	…	$55 + (n - 1)(-6)$
	symbols	$a_1 + 0 \cdot d$	$a_1 + 1 \cdot d$	$a_1 + 2 \cdot d$	$a_1 + 3 \cdot d$	…	$a_1 + (n - 1)d$

The following formula generalizes this pattern for any arithmetic sequence.

KEY CONCEPT ***n*th Term of an Arithmetic Sequence**

The nth term a_n of an arithmetic sequence with first term a_1 and common difference d is given by the following formula, where n is any positive integer.

$$a_n = a_1 + (n - 1)d$$

You can use the formula to find a term in a sequence given the first term and the common difference or given the first term and some successive terms.

Real-World Link

A hydraulic crane uses fluid to transmit forces from point to point. The brakes of a car use this same principle.

Source: howstuffworks.com

Real-World EXAMPLE **Find a Particular Term**

2 **CONSTRUCTION** **The table at the right shows typical costs for a construction company to rent a crane for one, two, three, or four months. If the sequence continues, how much would it cost to rent the crane for twelve months?**

Months	Cost ($)
1	75,000
2	90,000
3	105,000
4	120,000

Explore Since the difference between any two successive costs is \$15,000, the costs form an arithmetic sequence with common difference 15,000.

Plan You can use the formula for the nth term of an arithmetic sequence with $a_1 = 75{,}000$ and $d = 15{,}000$ to find a_{12}, the cost for twelve months.

Solve

$a_n = a_1 + (n - 1)d$ Formula for nth term

$a_{12} = 75{,}000 + (12 - 1)15{,}000$ $n = 12$, $a_1 = 75{,}000$, $d = 15{,}000$

$a_{12} = 240{,}000$ Simplify.

It would cost \$240,000 to rent the crane for twelve months.

Check You can find terms of the sequence by adding 15,000. a_5 through a_{12} are 135,000, 150,000, 165,000, 180,000, 195,000, 210,000, 225,000, and 240,000. Therefore, \$240,000 is correct.

CHECK Your Progress

2. The construction company has a budget of \$350,000 for crane rental. The job is expected to last 18 months. Will the company be able to afford the crane rental for the entire job? Explain.

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If you are given some of the terms of a sequence, you can use the formula for the nth term of a sequence to write an equation to help you find the nth term.

EXAMPLE 3 Write an Equation for the nth Term

Write an equation for the nth term of the arithmetic sequence 8, 17, 26, 35, ...

In this sequence, $a_1 = 8$ and $d = 9$. Use the nth term formula to write an equation.

$a_n = a_1 + (n - 1)d$ Formula for nth term

$a_n = 8 + (n - 1)9$ $a_1 = 8, d = 9$

$a_n = 8 + 9n - 9$ Distributive Property

$a_n = 9n - 1$ Simplify.

An equation is $a_n = 9n - 1$.

Study Tip

Checking Solutions

You can check to see that the equation you wrote to describe a sequence is correct by finding the first few terms of the sequence.

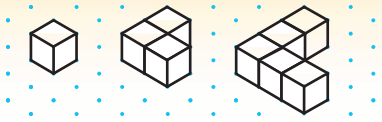
CHECK Your Progress

3. Write an equation for the nth term of the arithmetic sequence -1.5, -3.5, -5.5, ...

ALGEBRA LAB

Arithmetic Sequences

Study the figures below. The length of an edge of each cube is 1 centimeter.



MODEL AND ANALYZE

1. Based on the pattern, draw the fourth figure on a piece of isometric dot paper.

2. Find the volumes of the four figures.

3. Suppose the number of cubes in the pattern continues. Write an equation that gives the volume of Figure n.

4. What would the volume of the twelfth figure be?

Arithmetic Means Sometimes you are given two terms of a sequence, but they are not successive terms of that sequence. The terms between any two nonsuccessive terms of an arithmetic sequence are called **arithmetic means**. In the sequence below, 41, 52, and 63 are the three arithmetic means between 30 and 74.

19, 30, 41, 52, 63, 74, 85, 96, …

three arithmetic means between 30 and 74

The formula for the nth term of a sequence can be used to find arithmetic means between given terms of a sequence.

EXAMPLE Find Arithmetic Means

4 **Find the four arithmetic means between 16 and 91.**

Study Tip

Alternate Method

You may prefer this method. The four means will be $16 + d$, $16 + 2d$, $16 + 3d$, and $16 + 4d$. The common difference is $d = 91 - (16 + 4d)$ or $d = 15$.

You can use the nth term formula to find the common difference. In the sequence 16, __?__, __?__, __?__, __?__, 91, …, a_1 is 16 and a_6 is 91.

$a_n = a_1 + (n - 1)d$ Formula for the nth term

$a_6 = 16 + (6 - 1)d$ $n = 6, a_1 = 16$

$91 = 16 + 5d$ $a_6 = 91$

$75 = 5d$ Subtract 16 from each side.

$15 = d$ Divide each side by 5.

Now use the value of d to find the four arithmetic means.

$$16 \xrightarrow{+15} 31 \xrightarrow{+15} 46 \xrightarrow{+15} 61 \xrightarrow{+15} 76$$

The arithmetic means are 31, 46, 61, and 76. **CHECK** $76 + 15 = 91$ ✓

CHECK Your Progress

4. Find the three arithmetic means between 15.6 and 60.4.

CHECK Your Understanding

Example 1 (p. 622)

Find the next four terms of each arithmetic sequence.

1. 12, 16, 20, ...

2. 3, 1, −1, ...

Find the first five terms of each arithmetic sequence described.

3. $a_1 = 5, d = 3$

4. $a_1 = 14, d = -2$

5. $a_1 = \frac{1}{2}, d = \frac{1}{4}$

6. $a_1 = 0.5, d = -0.2$

Example 2 (p. 623)

7. Find a_{13} for the arithmetic sequence −17, −12, −7, ...

Find the indicated term of each arithmetic sequence.

8. $a_1 = 3, d = -5, n = 24$

9. $a_1 = -5, d = 7, n = 13$

10. $a_1 = -4, d = \frac{1}{3}, n = 8$

11. $a_1 = 6.6, d = 1.05, n = 32$

12. ENTERTAINMENT A basketball team has a halftime promotion where a fan gets to shoot a 3-pointer to try to win a jackpot. The jackpot starts at $5000 for the first game and increases $500 each time there is no winner. Ellis has tickets to the fifteenth game of the season. How much will the jackpot be for that game if no one wins by then?

Example 3 (p. 624)

13. Write an equation for the nth term of the arithmetic sequence −26, −15, −4, 7, ...

14. Complete: 68 is the __?__th term of the arithmetic sequence −2, 3, 8, ...

Example 4 (p. 625)

15. Find the three arithmetic means between 44 and 92.

16. Find the three arithmetic means between 2.5 and 12.5.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
17–26	1
27–34	2
35–40	3
41–44	4

Find the next four terms of each arithmetic sequence.

17. 9, 16, 23, ...
18. 31, 24, 17, ...
19. $-6, -2, 2, \ldots$
20. $-8, -5, -2, \ldots$

Find the first five terms of each arithmetic sequence described.

21. $a_1 = 2, d = 13$
22. $a_1 = 41, d = 5$
23. $a_1 = 6, d = -4$
24. $a_1 = 12, d = -3$

25. Find a_8 if $a_n = 4 + 3n$.
26. If $a_n = 1 - 5n$, what is a_{10}?

Find the indicated term of each arithmetic sequence.

27. $a_1 = 3, d = 7, n = 14$
28. $a_1 = -4, d = -9, n = 20$
29. $a_1 = 35, d = 3, n = 101$
30. $a_1 = 20, d = 4, n = 81$
31. a_{12} for $-17, -13, -9, \ldots$
32. a_{12} for $8, 3, -2, \ldots$

Real-World Link
Upon its completion in 1370, the Leaning Tower of Pisa leaned about 1.7 meters from vertical. Today, it leans about 5.2 meters from vertical.

Source: Associated Press

33. TOWER OF PISA To prove that objects of different weights fall at the same rate, Galileo dropped two objects with different weights from the Leaning Tower of Pisa in Italy. The objects hit the ground at the same time. When an object is dropped from a tall building, it falls about 16 feet in the first second, 48 feet in the second second, and 80 feet in the third second, regardless of its weight. How many feet would an object fall in the sixth second?

34. GEOLOGY Geologists estimate that the continents of Europe and North America are drifting apart at a rate of an average of 12 miles every 1 million years, or about 0.75 inch per year. If the continents continue to drift apart at that rate, how many inches will they drift in 50 years? (*Hint*: $a_1 = 0.75$)

Complete the statement for each arithmetic sequence.

35. 170 is the __?__ term of $-4, 2, 8, \ldots$.
36. 124 is the __?__ term of $-2, 5, 12, \ldots$.

Write an equation for the *n*th term of each arithmetic sequence.

37. 7, 16, 25, 34, ...
38. 18, 11, 4, −3, ...
39. $-3, -5, -7, -9, \ldots$
40. $-4, 1, 6, 11, \ldots$

Find the arithmetic means in each sequence.

41. 55, __?__, __?__, __?__, 115
42. 10, __?__, __?__, −8
43. −8, __?__, __?__, __?__, __?__, 7
44. 3, __?__, __?__, __?__, __?__, __?__, 27

Find the next four terms of each arithmetic sequence.

45. $\frac{1}{3}, 1, \frac{5}{3}, \ldots$
46. $\frac{18}{5}, \frac{16}{5}, \frac{14}{5}, \ldots$
47. 6.7, 6.3, 5.9, ...
48. 1.3, 3.8, 6.3, ...

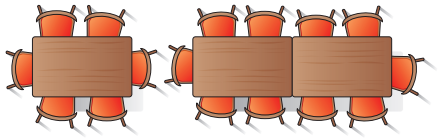
Find the first five terms of each arithmetic sequence described.

49. $a_1 = \frac{4}{3}, d = -\frac{1}{3}$
50. $a_1 = \frac{5}{8}, d = \frac{3}{8}$

51. VACATION DAYS Kono's employer gives him 1.5 vacation days for each month he works. If Kono has 11 days at the end of one year and takes no vacation time during the next year, how many days will he have at the end of that year?

52. DRIVING Olivia was driving her car at a speed of 65 miles per hour. To exit the highway, she began decelerating at a rate of 5 mph per second. How long did it take Olivia to come to a stop?

SEATING For Exercises 53–55, use the following information.
The rectangular tables in a reception hall are often placed end-to-end to form one long table. The diagrams below show the number of people who can sit at each of the table arrangements.



53. Make drawings to find the next three numbers as tables are added one at a time to the arrangement.

54. Write an equation representing the nth number in this pattern.

55. Is it possible to have seating for exactly 100 people with such an arrangement? Explain.

Find the indicated term of each arithmetic sequence.

56. $a_1 = 5, d = \frac{1}{3}, n = 12$

57. $a_1 = \frac{5}{2}, d = -\frac{3}{2}, n = 11$

58. a_{21} for 121, 118, 115, ...

59. a_{43} for 5, 9, 13, 17, ...

Use the given information to write an equation that represents the nth number in each arithmetic sequence.

60. The 15th term of the sequence is 66. The common difference is 4.

61. The 100th term of the sequence is 100. The common difference is 7.

62. The tenth term of the sequence is 84. The 21st term of the sequence is 161.

63. The 63rd term of the sequence is 237. The 90th term of the sequence is 75.

64. The 18th term of a sequence is 367. The 30th term of the sequence is 499. How many terms of this sequence are less than 1000?

EXTRA PRACTICE
See pages 914, 936.
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H.O.T. Problems

65. OPEN ENDED Write a real-life application that can be described by an arithmetic sequence with common difference −5.

66. REASONING Explain why the sequence 4, 5, 7, 10, 14, ... is not arithmetic.

67. CHALLENGE The numbers x, y, and z are the first three terms of an arithmetic sequence. Express z in terms of x and y.

68. *Writing in Math* Use the information on pages 622 and 623 to explain the relationship between n and a_n in the formula for the nth term of an arithmetic sequence. If n is the independent variable and a_n is the dependent variable, what kind of equation relates n and a_n? Explain what a_1 and d mean in the context of the graph.

STANDARDIZED TEST PRACTICE

69. ACT/SAT What is the first term in the arithmetic sequence?

$$\_\_\_\_, 8\tfrac{1}{3}, 7, 5\tfrac{2}{3}, 4\tfrac{1}{3}, \ldots$$

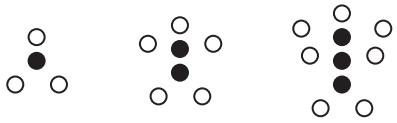
A 3

B $9\frac{2}{3}$

C $10\frac{1}{3}$

D 11

70. REVIEW The figures below show a pattern of filled circles and white circles that can be described by a relationship between 2 variables.



Which rule relates w, the number of white circles, to f, the number of dark circles?

F $w = 3f$

G $f = \frac{1}{2}w - 1$

H $w = 2f + 1$

J $f = \frac{1}{3}w$

Spiral Review

Find the exact solution(s) of each system of equations. (Lesson 10-7)

71. $x^2 + 2y^2 = 33$
$x^2 + y^2 - 19 = 2x$

72. $x^2 + 2y^2 = 33$
$x^2 - y^2 = 9$

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation. (Lesson 10-6)

73. $y^2 - 3x + 6y + 12 = 0$

74. $x^2 - 14x + 4 = 9y^2 - 36y$

75. If y varies directly as x and $y = 5$ when $x = 2$, find y when $x = 6$. (Lesson 8-4)

Simplify each expression. (Lesson 8-1)

76. $\dfrac{39a^3b^4}{13a^4b^3}$

77. $\dfrac{k+3}{5k\ell} \cdot \dfrac{10k\ell}{k+3}$

78. $\dfrac{5y - 15z}{42x^2} \div \dfrac{y - 3z}{14x}$

Find all the zeros of each function. (Lesson 6-8)

79. $f(x) = 8x^3 - 36x^2 + 22x + 21$

80. $g(x) = 12x^4 + 4x^3 - 3x^2 - x$

81. SAVINGS Mackenzie has \$57 in her bank account. She begins receiving a weekly allowance of \$15, of which she deposits 20% in her bank account. Write an equation that represents how much money is in Mackenzie's account after x weeks. (Lesson 2-4)

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression for the given values of the variable.** (Lesson 1-1)

82. $3n - 1; n = 1, 2, 3, 4$

83. $6 - j; j = 1, 2, 3, 4$

84. $4m + 7; m = 1, 2, 3, 4, 5$

85. $4 - 2k; k = 3, 4, 5, 6, 7$